PROBLEM XX.

In a Lottery conſiſting of 40000 *Tickets, among which are three particular Benefits, what is the Probability that taking* 8000 *of them, one or more of the particular Benefits ſhall be amongſt them.*

SOLUTION.

Firſt, In the Theorem belonging to the Remark of the foregoing Problem, having ſubſtituted reſpectively 8000, 40000, 32000, 3 and 1, in the room of c, n, d, a, and p; it will appear that the Probability of taking one preciſely of the three particular Benefits, will be $\frac{8000 \cdot 32000 \cdot 31999 \cdot 3}{40000 \cdot 39999 \cdot 39998} = \frac{48}{125}$ nearly.

Secondly, c, n, d, a being interpreted as before, let us ſuppoſe $p = 2$: hence the Probability of taking preciſely two of the particular Benefits will be found to be $\frac{8000 \cdot 7999 \cdot 32000 \cdot 3}{40000 \cdot 39999 \cdot 39998} = \frac{12}{125}$ nearly.

Thirdly, making $p = 3$, the Probability of taking all the three particular Benefits will be found to be $\frac{8000 \cdot 7999 \cdot 7998}{40000 \cdot 39999 \cdot 39998} = \frac{1}{125}$ very near.

Wherefore the Probability of taking one or more of the three particular Benefits will be $\frac{48+12+1}{125}$ or $\frac{61}{125}$ very near.

It is to be obſerved, that thoſe three Operations might have been contracted into one, by inquiring the Probability of not taking any of the three particular Benefits, which will be found to be $\frac{32000 \cdot 31999 \cdot 31998}{40000 \cdot 39999 \cdot 39998} = \frac{64}{125}$ nearly, which being ſubtracted from Unity, the remainder $1 - \frac{64}{125}$ or $\frac{61}{125}$ will ſhew the Probability required, and therefore the Odds againſt taking any of three particular Benefits will be 64 to 61 nearly.

PROBLEM XXI.

To find how many Tickets ought to be taken in a Lottery conſiſting of 40000, *among which are Three particular Benefits, to make it as probable that one or more of thoſe Three may be taken as not.*

SOLU-

SOLUTION.

Let the number of Tickets requiſite to be taken be $= x$, it will follow therefore from the Remark belonging to the xix^th^ Problem, that the Probability of not taking any of the particular Benefits will be $\frac{n-x}{n} \times \frac{n-x-1}{n-1} \times \frac{n-x-2}{n-2}$; but this Probability is equal to $\frac{1}{2}$, ſince by Hypotheſis the Probability of taking one or more of them is equal to $\frac{1}{2}$, from whence we ſhall have the Equation $\frac{n-x}{n} \times \frac{n-x-1}{n-1} \times \frac{n-x-2}{n-2} = \frac{1}{2}$, which Equation being ſolved, the Value of x will be found to be nearly 8252.

N. B. The Factors whereof both the Numerator and Denominator are compoſed, being but few, and in arithmetic progreſſion; and beſides, the difference being very ſmall in reſpect of n; thoſe Terms may be conſidered as being in geometric Progreſſion: wherefore the Cube of the middle Term $\frac{n-x-1}{n-1}$, may be ſuppoſed equal to the product of the Multiplication of thoſe Terms; from whence will ariſe the Equation $\frac{\overline{n-x-1}^{3}}{\overline{n-1}^{3}} = \frac{1}{2}$, or neglecting Unity in both Numerator and Denominator, $\frac{\overline{n-x}^{3}}{n^3} = \frac{1}{2}$ and conſequently x will be found to be $= n \times 1 - \sqrt[3]{\frac{1}{2}}$ or $n \times 1 - \frac{1}{2}\sqrt[3]{4}$, but $n = 40000$, and $1 - \frac{1}{2}\sqrt[3]{4} = 0.2063$; wherefore $x = 8252$.

In the Remark belonging to the ſecond Problem, a Rule was given for finding the number of Tickets that were to be taken to make it as probable, that one or more of the Benefits would be taken as not; but in that Rule it was ſuppoſed, that the proportion of the Blanks to the Prizes was often repeated, as it uſually is in Lotteries: now in the caſe of the preſent Problem, the particular Benefits being but three in all, the remaining Tickets are to be conſidered as Blanks in reſpect of them; from whence it follows, that the proportion of the number of Blanks to one Prize being very near as 13332 to 1, and that proportion being repeated but three times in the whole number of Tickets, the Rule there given would not have been ſufficiently exact, for which reaſon it was thought neceſſary to give another Rule in this place.

PROBLEM

PROBLEM XXII.

Suppoſing a Lottery of 100000 *Tickets, whereof* 90000 *are Blanks, and* 10000 *are Benefits, to determine accurately what the odds are of taking or not taking a Benefit, in any number of Tickets aſſigned.*

SOLUTION.

Suppoſe the number of Tickets to be 6; then let us inquire into the Probability of taking no Prize in 6 Tickets, which to find, let us make uſe of the Theorem ſet down in the Corollary of the xix^th^ *Problem*, wherein it will appear that the number of Chances for taking no Prize in 6 Tickets, making $a = 10000$, $b = 90000$, $c = 6$, $p = 0$, $n = 100000$, will be

$$\frac{90000}{1} \times \frac{89999}{2} \times \frac{89998}{3} \times \frac{89997}{4} \times \frac{89996}{5} \times \frac{89995}{6},$$

and that the whole number of Chances will be

$$\frac{100000}{1} \times \frac{99999}{2} \times \frac{99998}{3} \times \frac{99997}{4} \times \frac{99996}{5} \times \frac{99995}{6};$$ then

dividing the firſt number of Chances by the ſecond, which may eaſily be done by Logarithms, the Quotient will be 0.53143, and this ſhews the Probability of taking no Prize in 6 Tickets, and this decimal fraction being ſubtracted from Unity, the Remainder 0.46857 ſhews the Probability of taking one Prize or more in 6 Tickets; wherefore the Odds againſt taking any Prize in 6 Tickets, will be 53143 to 46857.

If we ſuppoſe now that the number of Tickets taken is 7, then carrying each number of Chances above-written one ſtep farther, we ſhall find that the Probability of taking no Prize in 7 Tickets is 0.47828, which fraction being ſubtracted from Unity, the remainder will be 0.52172, which ſhews the Odds of taking one Prize or more in 7 Tickets to be 52172 to 47828.

REMARK.

When the number of Tickets taken bear a very inconſiderable proportion to the whole number of Tickets, as it happens in the caſe of this Problem, the Queſtion may be reſolved as a Problem depending on the Caſt of a Die; we may therefore ſuppoſe a Die of

of 10 Faces having one of its Faces such as the Ace representing a Benefit, and all the other nine representing Blanks, and inquire into the Probability of missing the Ace 6 times together, which by the Rules given in the Introduction, will be found to be $\frac{9^6}{10^6} = 0.53144$ differing from what we had found before but one Unit in the fifth place of Decimals: now if we inquire into the Probability of missing the Ace 7 times, we shall find it 0.47829 differing also but one Unit in the fifth of Decimals, from what had been found before, and therefore in such cases as this we may use both Methods indifferently, but the first will be exact if we actually multiply the numbers together, the second is only an approximation.

But both Methods confirm the truth of the practical Rule given in our third Problem, about finding what number of Tickets are necessary for the equal Chance of a Prize; for multiplying as it is there directed, the number 9 representing the Blanks by 0.7, the Product 6.3 will shew that the number requisite is between 6 and 7.

PROBLEM XXIII.

The same things being given as in the preceding Problem, suppose the price of each Ticket to be 10 *L. and that after the Lottery is drawn,* 7 *L.* — 10 *sh. be returned to the Blanks, to find in this Lottery the value of the Chance of a Prize.*

SOLUTION.

There being 90000 Blanks, to every one of which 7 *L.* — 10 *sh.* is returned, the total Value of the Blanks is 675000 *L.* and consequently the total Value of the Benefits is 325000 *L.* which being divided by 10000 number of the Benefits, the Quotient is 32 *L.* — 10 *sh.* and therefore one might for the Sum of 32 *L.* — 10 *sh.* be intitled to have a Benefit certain, taken at random out of the whole number of Benefits: the Purchaser of a Chance has therefore 1 Chance in 10 for the Sum of 32 *L.* — 10 *sh.* and 9 Chances in 10 for losing his Money; from whence it follows, that the value of his Chance is the 10th part of 32 *L.* — 10 *sh.* that is 3 *L.* — 5 *sh.* And therefore the Purchaser of a Chance, by giving the Seller 3 *L.* — 5 *sh.* is intitled to the Chance of a Benefit, and ought not to return any thing to the Seller, altho' he should have a Prize; for

the ſeller having $3^{L.} - 5^{ſh.}$ ſure, and 9 Chances in 10 for $7^{L} - 10^{ſh.}$ the Value of which Chances is $6^{L.} - 15^{ſh.}$ it follows that he has his $10^{L.}$

PROBLEM XXIV.

Suppoſing ſtill the ſame Lottery as has been mentioned in the two preceding Problems, let A *engage to furniſh* B *with a Chance, on condition that whenever the Ticket on which the Chance depends, ſhall happen to be drawn, whether it proves a Blank or a Prize,* A *ſhall furniſh* B *with a new Chance, and ſo on, as often as there is occaſion, till the whole Lottery be drawn; to find what conſideration* B *ought to give* A *before the Lottery begins to be drawn, for the Chance or Chances of one or more Prizes, admitting that the Lottery will be* 40 *days a drawing.*

SOLUTION.

Let $3^{L.} - 5^{ſh.}$ which is the abſolute Value of a Chance be called *s*.

1°. *A* who is the Seller ought to conſider, that the firſt Day, he furniſhes neceſſarily a Chance whoſe Value is *s*.

2°. That the ſecond day; he does not neceſſarily furniſh a Chance, but conditionally, *viz.* if it ſo happen that the Ticket on which the Chance depends, ſhould be drawn on the firſt day; but the Probability of its being drawn on the firſt day is $\frac{1}{40}$; and therefore he ought to take $\frac{1}{40}s$ for the conſideration of the ſecond day.

3°. That in the ſame manner, he does not neceſſarily furniſh a Chance on the third day, but conditionally in caſe the only Ticket depending (for there can be but one) ſhould happen to be drawn on the ſecond day, of which the Probability being $\frac{1}{39}$, by reaſon of the remaining 39 days from the ſecond incluſive to the laſt, it follows, that the Value of that Chance is $\frac{1}{39}s$.

4°. And for the ſame reaſon, the Value of the next is $\frac{1}{38}s$. and ſo on.

The Purchaſer ought therefore to give the Seller

$1 + \frac{1}{40} + \frac{1}{39} + \frac{1}{38} + \frac{1}{37} \ldots\ldots\ldots + \frac{1}{2}$, the whole multiply'd by s, or

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \ldots\ldots + \frac{1}{40}$, the whole multiply'd by s. Now it being pretty laborious to ſum up thoſe 40 Terms, I have here made uſe of a Rule which I have given in the Supplement to my *Miſcellanea Analytica*, whereby one may in a very ſhort time ſum up as many of thoſe Terms as one pleaſes, tho' they were 10000 or more, and by that Rule, the Sum of thoſe 40 Terms will be found to be 4.2785 very near, which being multiply'd by s which in this caſe is 3.25, the product 13.9 will ſhew that the Purchaſer ought to give the Seller about $13^{L.} - 18^{ſh.}$

COROLLARY.

The Value of the Chance s for one ſingle day that ſhall be fixed upon, is the Value of that Chance divided by the number of Days intercepted between that Day incluſive, and the number of Days remaining to the end of the Lottery: which however muſt be underſtood with that reſtriction, that the Day fixed upon muſt be choſe before the Lottery begins, or that if it be done on any other Day, the State of the Lottery muſt be known, and a new Calculation made accordingly for the Value of s.

PROBLEM XXV.

To find the Probability of taking four Hearts, three Diamonds, two Spades, and one Club in ten Cards out of a Stock containing thirty-two.

SOLUTION.

Firſt, The number of Chances for taking four Hearts out of the whole number of Hearts that are in the Stock, that is out of Eight, will be $\frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} = 70$.

Secondly, The number of Chances for taking three Diamonds out of Eight, will be $\frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3} = 56$.

Thirdly,

Thirdly, The number of Chances for taking two Spades out of Eight, will be $\frac{8 \cdot 7}{1 \cdot 2} = 28$.

Fourthly, The number of Chances for taking one Club out of Eight, will be $\frac{8}{1} = 8$.

And therefore multiplying all thoſe particular Chances together, the product $70 \times 56 \times 28 \times 8 = 878080$ will denote the whole number of Chances for taking four Hearts, three Diamonds, two Spades, and one Club.

Fifthly, The whole number of Chances for taking any ten Cards out of thirty-two is

$$\frac{32 \cdot 31 \cdot 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10} = 64512240$$

And therefore dividing the firſt Product by the ſecond, the quotient $\frac{878080}{64512240}$ or $\frac{1}{75}$ nearly, will expreſs the Probability required; from which it follows, that the Odds againſt taking four Hearts, three Diamonds, two Spades, and one Club in ten Cards, out of a Stock containing thirty-two, are very near 74 to 1.

Of the Game of QUADRILLE.

PROBLEM XXVI.

The Player having 3 Matadors and three other Trumps by the loweſt Cards in black or red, what is the Probability of his forcing all the Trumps.

SOLUTION.

In order to ſolve this Problem, it is to be conſidered, that the Player whom I call *A* forces the Trumps neceſſarily, if none of the other Players whom I call *B*, *C*, *D* has more than three Trumps; and therefore, if we calculate the Probability of any one of them having more than three Trumps, which caſe is wholly againſt *A*, we may from thence deduce what will be favourable to him, but let us firſt ſuppoſe that he plays in black.

Since the number of Trumps in black is 11, and that *A* by ſuppoſition has 6 of them, then the number of Trumps remaining amongſt *B*, *C*, *D* is 5; and again, ſince the number of all the other remaining Cards which we may call Blanks is 29, whereof *A* has

4, it follows that there are 25 Blanks amongſt *B*, *C*, *D*, and therefore the number of Chances for *B* in his 10 Cards to have 4 Trumps and 6 Blanks, is by the Corollary of the xix^th^ Problem

$$\frac{5\times4\times3\times2}{1\cdot2\cdot3\cdot4}\times\frac{25\cdot24\cdot23\cdot22\cdot21\cdot20}{1\cdot2\cdot3\cdot4\cdot5\cdot6}$$

And likewiſe the number of Chances for his having 5 Trumps and 5 Blanks, is by the ſame Corollary.

$$\frac{5\cdot4\cdot3\cdot2\cdot1}{1\cdot2\cdot3\cdot4\cdot5}\times\frac{25\cdot24\cdot23\cdot22\cdot21}{1\cdot2\cdot3\cdot4\cdot5}$$

And therefore the number of all the Chances of *B* againſt *A* is $106\times5\times7\times11\times23$; but the number of Chances whereby any 10 Cards may be taken out of 30 is $\frac{30\cdot29\cdot28\cdot27\cdot26\cdot25\cdot24\cdot23\cdot22\cdot21}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10}$. which being reduced to $5\times7\times9\times11\times13\times23\times29$, it follows that the Probability of *B*'s having more than three Trumps is $\frac{106\times5\cdot7\cdot11\cdot23}{3\cdot5\cdot7\cdot11\cdot13\cdot23\cdot29}=\frac{106}{9\times13\cdot29}$: but this Probability falls as well upon *C* and *D* as upon *A*, and therefore it ought to be multiply'd by 3, which will make it $\frac{106}{3\times13\times29}=\frac{106}{1131}$; and this being ſubtracted from Unity, the remainder $\frac{1025}{1131}$ will expreſs the Probability of *A*'s forcing all the Trumps, and therefore the Odds of his forcing the Trumps are 1025 againſt 106, that is 29 to 3 nearly.

But if *A* plays the ſame Game in red, his advantage will be conſiderably leſs than before, for there being 12 Trumps in red, whereof he has 6, *B* may have 4, or 5, or 6 of them, ſo that the number of the Chances which *B* has for more than three Trumps will be reſpectively as follows:

$$\frac{6\times5\times4\cdot3}{1\cdot2\cdot3\cdot4}\times\frac{24\cdot23\cdot22\cdot21\cdot20\cdot19}{1\cdot2\cdot3\cdot4\cdot5\cdot6}$$
$$\frac{6\cdot5\cdot4\cdot3\cdot2}{1\cdot2\cdot3\cdot4\cdot5}\times\frac{24\cdot23\cdot22\cdot21\cdot20}{1\cdot2\cdot3\cdot4\cdot5}$$
$$\frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{1\cdot2\cdot3\cdot4\cdot5\cdot6}\times\frac{24\cdot23\cdot22\cdot21}{1\cdot2\cdot3\cdot4}$$

Now the Sum of all thoſe Chances being $215\times23\times22\times21$, and the Sum of all the Chances for taking any 10 Cards out of 30, being $5\times7\times9\times11\times13\times23\times29$, as appears by the preceding caſe, it follows, that the Probability of *B*'s having more than three Trumps is $\frac{215\cdot23\cdot22\cdot21}{5\cdot7\cdot9\cdot11\cdot13\cdot23\cdot29}=\frac{86}{3\cdot13\cdot29}$, but this Probability

lity falling as well upon *C* and *D*, as upon *B*, ought to be multiply'd by 3, which will make it $\frac{86}{13 \cdot 29} = \frac{86}{377}$; and this being ſubtracted from Unity, the remainder $\frac{291}{377}$, will expreſs the Probability of *A*'s forcing all the Trumps, and therefore the Odds of his forcing all the Trumps is in this caſe 291 to 86, that is nearly 10 to 3.

PROBLEM XXVII.

The Player A *having Spadille, Manille, King, Queen, and two ſmall Trumps in black, to find the Probability of his forcing all the Trumps.*

SOLUTION.

A forces the Trumps neceſſarily, if *Baſte* accompanied with two other Trumps be not in one of the Hands of *B*, *C*, *D*, and as *Baſte* ought to be in ſome Hand, it is indifferent where we place it; let it therefore be ſuppoſed that *B* has it, in conſequence of which let us conſider the number of Chances for his having beſides *Baſte*,

1°. 2 Trumps and 7 Blanks.
2°. 3 Trumps and 6 Blanks.
3°. 4 Trumps and 5 Blanks.

Now the Blanks being in all 29, whereof *A* has 4, it follows that the number of remaining Blanks is 25, and the number of Trumps being in all 11, whereof *A* has 6 by Hypotheſis, and *B* has 1, *viz. Baſte*, it follows that the number of remaining Trumps is 4; and therefore the Chances which *B* has againſt the Player are reſpectively as follows:

$$\frac{4 \cdot 3}{1 \cdot 2} \times \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7}$$

$$\frac{4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3} \times \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}$$

$$\frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}$$

The Sum of all which is $1441 \times 5 \times 23 \times 22$; but the Sum of all the Chances whereby *B* may join any 9 Cards to the *Baſte* which he has already is $\frac{29 \times 28 \times 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23 \times 22 \cdot 21}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9} = 23 \times 7 \times 3 \times 13 \times 5 \times 23 \times 11$: and therefore the Probability of *Baſte* being in one

one Hand, accompany'd with two Trumps at leaſt, is expreſſed by the Fraction $\frac{1441 \cdot 5 \cdot 23 \cdot 22}{29 \cdot 7 \cdot 3 \cdot 13 \cdot 5 \cdot 23 \cdot 11} = \frac{131 \cdot 22}{29 \cdot 7 \cdot 3 \cdot 13} = \frac{2882}{7917}$ and this being ſubtracted from Unity, the remainder will be $\frac{5035}{7917}$, and therefore the Odds of *A*'s forcing the Trumps are 5035 to 2882, which are very near 7 to 4.

But if it be in red, *A* has the ſmall diſadvantage of 19703 againſt 19882, or nearly 110 againſt 111.

PROBLEM XXVIII.

The Player having Spadille, Manille, and 5 other Trumps more by the loweſt in red, what is the Probability, by playing Spadille and Manille, of his forcing 4 Trumps.

SOLUTION.

The 5 remaining Trumps being between *B*, *C*, *D*, their various diſpoſitions are the following:

B,	*C*,	*D*
1,	2,	2
2,	3,	0
3,	1,	1
4,	1,	0
5,	0,	0

Which muſt be underſtood in ſuch manner, that what is here aſſigned to *B* may as well belong to *C* or *D*.

Now it is plain, that out of thoſe five diſpoſitions there are only the two firſt that are favourable to *A*, let us therefore ſee what is the Probability of the firſt diſpoſition.

The number of Chances of *B* to have 1 Trump and 9 Blanks are $\frac{5}{1} \times \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9} = 5 \times 5 \times 5 \times 11 \times 17 \times 19 \times 23$, but the number of all the Chances whereby he may take any 10 Cards out of 30 is $5 \times 7 \times 9 \times 11 \times 13 \times 23 \times 29$ as has been ſeen already in one of the preceding Problems, and therefore the Probability of *B*'s having one Trump and nine Blanks is $\frac{5 \cdot 5 \cdot 5 \cdot 11 \cdot 17 \cdot 19 \cdot 23}{5 \cdot 7 \cdot 9 \cdot 11 \cdot 13 \cdot 23 \cdot 29} = \frac{25 \cdot 19 \cdot 17}{7 \cdot 9 \cdot 13 \cdot 29}$.

Now

Now in order to find the number of Chances for *C* to have 2 Trumps and 8 Blanks, it must be considered that *A* having 6 Trumps, and *B* 1, the number of remaining Trumps is 4, and likewise that *A* having 4 Blanks, and *B* 9, the number of remaining Blanks is 15, and therefore that the number of Chances for *C* to have 2 Trumps and 8 Blanks is

$$\frac{4 \cdot 3}{1 \cdot 2} \times \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8} = 6 \times 9 \times 10 \times 11 \times 13$$

But the number of all the Chances whereby *C* may take any 10 Cards out of 20 remaining between him and *D* is

$$\frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10} = 4 \times 11 \times 13 \times 17 \times 19,$$

and therefore the Probability of *C*'s having 2 Trumps and 8 Blanks is

$$\frac{6 \cdot 9 \cdot 10 \cdot 11 \cdot 13}{4 \cdot 11 \cdot 13 \cdot 17 \cdot 19} = \frac{6 \cdot 9 \cdot 10}{4 \cdot 17 \cdot 19} = \frac{9 \cdot 15}{17 \cdot 19}.$$

Now *A* being supposed to have had 6 Trumps, *B* 1, and *C* 2, *D* must have 2 necessarily, and therefore no new Calculation ought to be made on account of *D*. It follows therefore that the Probability of the disposition 1, 2, 2, belonging respectively to *B*, *C*, *D* ought to be express'd by $\frac{25 \cdot 19 \cdot 17}{7 \cdot 9 \cdot 13 \cdot 29} \times \frac{9 \cdot 15}{17 \cdot 19} = \frac{15 \cdot 25}{7 \cdot 13 \cdot 29}$.

Now three things, whereof two are alike, being to be permuted 3 different ways, it follows that the Probability of the Disposition 1, 2, 2, as it may happen in any order will be $\frac{3 \cdot 15 \cdot 25}{7 \cdot 13 \cdot 29} = \frac{1125}{2639}$.

It will be found in the same manner, that the Probability of the Disposition 2, 3, 0 as it belongs respectively to *B*, *C*, *D*, is $\frac{2 \cdot 5 \cdot 10}{7 \cdot 13 \cdot 29}$; but the number of Permutations of three things which are all unlike being 6, it follows that the said Probability ought to be multiply'd by 6, which will make it $\frac{6 \cdot 2 \cdot 5 \cdot 10}{7 \cdot 13 \cdot 29} = \frac{600}{2639}$.

From all which it follows, that the Probability of *A*'s forcing 4 Trumps is $\frac{1125 + 600}{2639} = \frac{1725}{2639}$, which fraction being subtracted from Unity, the remainder will be $\frac{914}{2639}$, and therefore the Odds of *A*'s forcing 4 Trumps are 1725 to 914, that is very near 17 to 9.

PROBLEM

PROBLEM XXIX.

A *having* 4 *Matadors, in Diamonds, with the two black Kings, each accompanied with two ſmall Cards of their own colour; what is the Probability that no one of the others* B, C, D *has more than* 4 *Trumps, or in caſe he has more, that he has alſo of the colour of both his Kings, in which caſes he wins neceſſarily.*

SOLUTION.

The Chances that are againſt *A* are as follows, it being poſſible that *B* may have

Diamonds,	Hearts,	Number of Chances.
5,	5	14112
6,	4	5880
7,	3	960
8,	2	45
	Sum	20997

Diamonds,	Spades,	Hearts,	Number of Chances.
5,	1,	4	70560
5,	2,	3	100800
5,	3,	2	50400
5,	4,	1	8400
5,	5,	0	336
6,	1,	3	20160
6,	2,	2	18900
6,	3,	1	5600
6,	4,	0	420
7,	1,	2	2160
7,	2,	1	1200
7,	3,	0	160
8,	1,	1	60
8,	2,	0	15
			279171

Now

Now by reaſon that among *B*, *C*, *D*, there are as many Clubs as Spades, *viz.* 6 of each ſort, it follows that the Clubs may be ſubſtituted in the room of the Spades, which will double this laſt number of Chances, and make it 558330; and therefore, adding together the firſt and ſecond number of Chances, the Sum will be 579327, which will be the whole number of Chances, whereby *B* may withſtand the Expectation of *A*, but the number of all the Chances which *B* has for taking any 10 Cards out of 30 is $5\times7\times9\times11\times13\times23\times29 = 30045015$; from which it follows that the Probability of *B*'s withſtanding the Expectation of *A* is $\frac{579327}{30045015}$, but as this may fall as well upon *C* and *D* as upon *B*, it follows that this Probability ought to be multiply'd by 3, then the Product $\frac{1737981}{30045015}$ will expreſs the Probability of *A*'s loſing, and this being ſubtracted from Unity, the remainder will expreſs the Probability of *A*'s winning, and therefore the Odds of *A*'s winning will be little more than $16\frac{1}{4}$ to 1.

PROBLEM XXX.

A *having Spadille, Manille, King, Knave, and two other ſmall Trumps in black, what is the Probability that Baſte accompany'd with two other Trumps, or the Queen accompany'd with three other ſhall not be in the ſame hand, in which caſe* A *wins neceſſarily?*

SOLUTION.

The Probability of Baſte being in one hand, accompany'd with two other Trumps, has been found in Problem xxvii to be $\frac{2882}{7917}$.

The number of Chances for him who has the Queen, to have alſo three other Trumps, excluding Baſte is

$$\frac{3\cdot2\cdot1}{1\cdot2\cdot3}\times\frac{25\cdot24\cdot23\cdot22\cdot21\cdot20}{1\cdot2\cdot3\cdot4\cdot5\cdot6} = 5\times7\times11\times20\times23$$

but the number of Chances for joining any 9 Cards to the Queen is $3\times5\times7\times11\times13\times23\times29$, and therefore the Probability of the Queen's being in one hand, accompany'd with three other Trumps is

$$\frac{5\cdot7\cdot11\cdot20\cdot23}{3\times5\cdot7\cdot11\cdot13\cdot23\cdot29} = \frac{20}{3\cdot13\cdot29} = \frac{20}{1131} = \frac{140}{7917},$$

now this Probability being added to the former, the Sum will be $\frac{3022}{7917}$, and therefore the Odds of *A's* not being withſtood either from Baſte being accompany'd with two other Trumps, or from the Queen accompany'd with three, are 4895 to 3022, nearly as 13 to 8.

It may be obſerved, that the reaſon of Baſte being excluded from among the Trumps that accompany the Queen is this, if the Queen be accompany'd with Baſte and two other Trumps, the Baſte itſelf is accompany'd with three Trumps, which caſe had been taken in already in the firſt part of this Solution.

PROBLEM XXXI.

A *having three Matadors in Spades with the Kings of Hearts, Diamonds, and Clubs, two ſmall Hearts, two ſmall Diamonds, and one ſmall Club, to find the Probability that not above three Spades ſhall be in one hand, or that, if there be above three, there ſhall be alſo of the colour of the three Kings, in which caſe* A *wins neceſſarily.*

SOLUTION.

1°. The Probability of not above three Trumps being in one hand $=0.332141$.

The Probability that one of the Oppoſers ſhall have 4 Trumps, and at the ſame time Hearts, Diamonds, and Clubs, and that no other ſhall have 4 Trumps is $=0.393501$.

The Probability that two of the Oppoſers ſhall have 4 Trumps, and at the ſame time Hearts, Diamonds and Clubs is $=0.013836$.

The Probability that one of the Oppoſers ſhall have 5 Trumps, and at the ſame time Hearts, Diamonds and Clubs is $=0.103019$.

The Probability that one of the Oppoſers ſhall have 6 Trumps, Hearts, Diamonds and Clubs is $=0.001041$.

The Probability of one of the Oppoſers having 7 Trumps, and at the ſame time Hearts, Diamonds and Clubs is $=0.000313$.

Now the Sum of all theſe Probabilities is 0.843851, which being ſubtracted from Unity, the remainder is 0.156149, and therefore the Odds of the Players winning are as 843851 to 156149, that is very near as 27 to 5.

PROBLEM